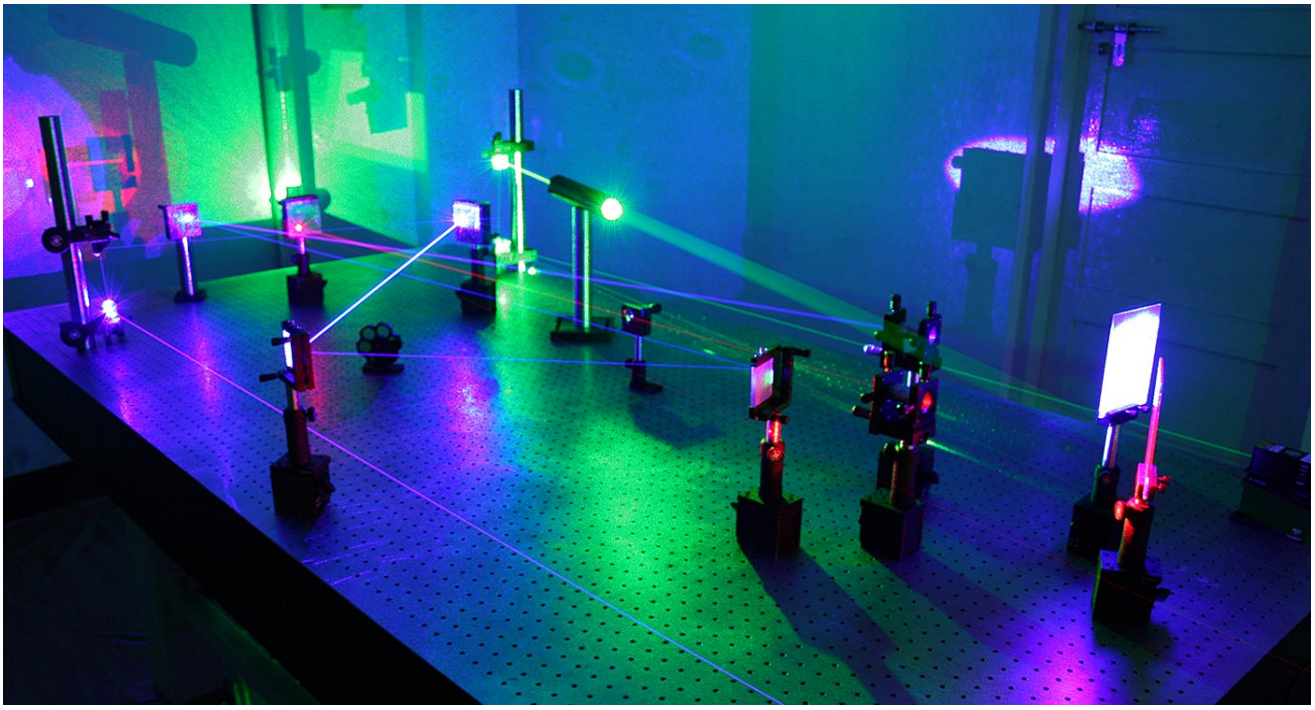
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Digital Holography



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Date: March 11, 2026

EPFL

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***Abstract*—This report summarizes the experimental results obtained to gain theoretical understanding and practical knowledge of Digital Holography. The methodology to isolate the information is explained and demonstrated with experimental graphs. The thickness of the layer of the object is found to be around $0.8\ \mu m$. The behaviour of applying mask on a SLM is also studied and the angular separation is found to be $\theta = 3.72°$. The periodic pattern is due to the pixel period $8\ \mu m$. The variation of the focal length is also analyzed and it shows that multiple holograms can be embedded in one beam (using BPM). A study of the transmission matrix is also conducted to characterize a optic fiber. The effects of bending this fiber are also developed. The number of modes in the fiber is 1054 and the information stored in the matrix is 16864 bytes.**

I. Introduction

Digital holography has many applications in various domains of sciences. For example, it is used to detect the presence of microscopic transparent cells in biology or to observe very small stresses and deformations in material sciences.

Holography can be compared to photography because they both record the amplitude of a field but holography can also obtain the phase. This allows applications that are not possible with a simple camera.

Holography consists in the acquisition and reproduction of optical waves. Note that what we call hologram is a transparent object in which an optical wave is stored. Let us consider a monochromatic plane in the a plane Π such that $(x, y, 0) \in \Pi$. This wave has a complex amplitude of $U_o(x, y)$ in Π. One stores in a thin film a complex transmittance $t(x, y)$, then one can recover the original wave by sending on this film a plane wave of amplitude 1. In this case, in Π plane, we have $U(x, y) = 1 \cdot t(x, y) = U_o(x, y)$. Then for all points $(x, y, z) \mid z > 0$, the reconstructed wave is equal to the original one. (see Figure 1)

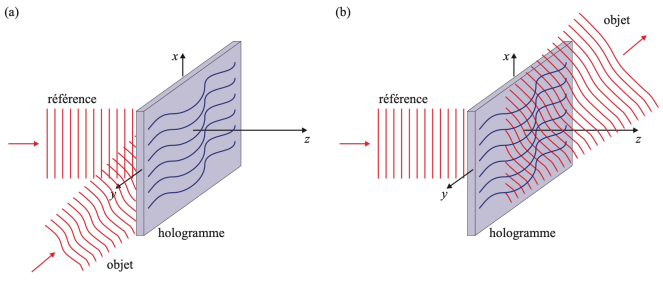

Fig. 1. (a) Storage and (b) Recovery of an object wave U_o with a reference U_r [3]

The main key is to store a complex transmission in a thin film which would allow to store the interference figure between U_o an object wave, and U_r the reference one.

II. Task 1: Answer the following questions

The purpose of this section is to develop theoretical background for digital holography by answering some questions.

- The camera sensor detects the sum of the object wave O and the reference wave R. Write the equation for the intensity I obtained on the camera sensor. Show the appearance of four terms. Briefly describe the meaning of each term. For this calculation, note that R and O are complex quantities.

As explained in the question, the camera detects the addition of two waves (R and O). The wave can then be expressed as $W = R + O$. The intensity of a wave is found as follows.

$$\begin{aligned} I_W(x,y) =& |W|^2 = WW^* = (R+O)(R^*+O^*) \\ =& RR^* + OO^* + RO^* + R^*O \\ =& I_R + I_O + \mathbb{R}\{RO^*\} + \mathbb{R}\{R^*O\} \end{aligned} \tag{1}$$

The intensity of both the reference and the object waves is found in the expression of the total intensity but there are two new terms. These corresponds to interference terms between the two waves.

- Set the reference wave equal to an off-axis plane wave: $R(x, y) = e^{jk_0(\sin(\theta_x)x + \sin(\theta_y)y)}$. Here, the magnitude of the wave vector is $k_0 = \frac{2\pi}{\lambda_0}$ and the angles of this wave with the x- and y- axis respectively are θ_x and θ_y. Note that we neglected the time dependence by setting $t = 0$, and we also assumed the camera plane is at $z = 0$. Assume a general form for the object wave: $O = O(x, y)$. Now plug these waves in the expression you found in the previous question and take the two-dimensional Fourier transform. Determine the location of each term in frequency space by using the Fourier shift property:

By plugging the expressions given in Equation (1), the following development can be done. Note that the intensity of $R(x, y)$ is one as the amplitude of the wave is one.

$$I_W(x,y) = 1 + I_O + RO^* \ + R^*O \tag{2}$$

With the two last terms developed here under. Note that there is no information on $O(x, y)$.

$$RO^* \ = O^* e^{jk_0(\sin(\theta_x)x + \sin(\theta_y)y)} \tag{3}$$

$$R^*O \ = Oe^{-jk_0(\sin(\theta_x)x + \sin(\theta_y)y)} \tag{4}$$

By applying Fourier's transform and using some properties of this application, the reasoning can be brought further. The property of Fourier is shift and it is described in Equation (5).

$$\mathbb{F}\{e^{j(ax+by)}f(x,y)\}(k_x,k_y) = \hat{f}(k_x-a,k_y-b) \quad (5)$$

The two waves intensities I_R and I_O will become simple Dirac delta's as they are not multiplied by any exponential. The two other terms will be shifted.

$$\begin{aligned}\mathbb{F}\{I(x,y)\} =& I_O\delta(k_x,k_y) + \delta(k_x,k_y)\\ =& \hat{O^*}(k_x - k_0\sin\theta_x, k_y - k_0\sin\theta_x) \quad (6)\\ =& \hat{O}(k_x + k_0\sin\theta_x, k_y + k_0\sin\theta_x)\end{aligned}$$

Based on this last equation, some assumption can be made on the output. There will always be information in the center of the plane due to the intensity of both waves. The interferences are shifted by values depending on the angle θ_x and θ_y.

- Assume further, for simplicity, that the reference wave has an angle only in x (i.e. $\theta_y = 0$), and the object wave is a plane wave coming in at perpendicular angle on the camera sensor,i.e. $O(x,y) = 1$. The sum of O and R creates a periodic intensity pattern on the camera. What is the spacing d between the oscillations (period) of this pattern?

Under these conditions, the formula for the intensity can be simplified into the form of Equation (7), where the first two terms are the intensities I_R and I_O and the third term is the sum of the two real parts (using $\cos(-x) = \cos(x)$).

$$I = e^{2jk_0\sin\theta_x x} + 1 + 2\cos(k_0\sin\theta_x x) \quad (7)$$

Equation (7) shows I oscillating along x due to the cosine term. Knowing that the cosine is 2π periodic, the relation (11) can be deduced for $x = d$.

$$k_0\sin\theta_x d = 2\pi \iff d = \frac{\lambda_0}{\sin(\theta_x)} \quad (8)$$

- A camera with a pixel size of L_{pixel} can correctly resolve a pattern only if its spatial period d is larger than two pixels: $d > 2L_{pixel}$. This is the Nyquist-Shannon sampling criterion. It is schematically illustrated in Figure 2. Under this condition, what is the maximum allowable angle θ_x of the off-axis reference beam? Calculate this angle in degrees for a wavelength of $532\ nm$ and a camera with $8\ \mu m$ pixels.

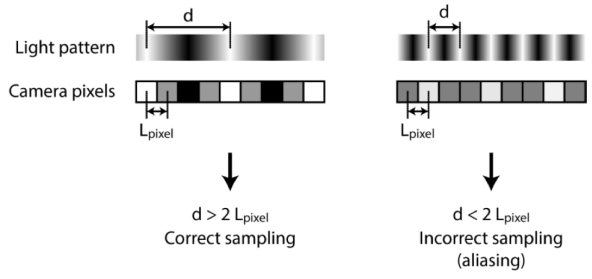

Fig. 2. Sampling of a periodic fringe pattern with a pixelated device

Having $d = 2L_{pixel}$ and therefore taking the limit case of the Nyquist-Shannon criterion, the maximal angle θ_x can be found. This is done by substituting the variables in equation (9) and solving for θ_x. Note that $L_{pixel} = 8\ \mu m$ and $\lambda_0 = 532\ nm$.

$$\theta_x = \arcsin\frac{\lambda_0}{2L_{pixel}} = 1.91° \quad (9)$$

The angle θ_x must be smaller than $1.91°$ for the system to work properly.

III. Task 2: Record an off-axis digital hologram

This task aims to get an off-axis digital hologram, which will be then used as data in Task 3.

A. Setup

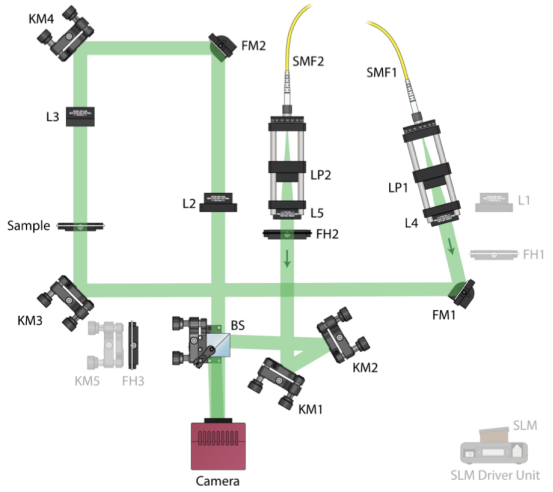

Fig. 3. Setup for the digital holography experiments [1]

The experimental setup for the digital holography experiments is shown in Figure 3. Laser light is brought to the setup by the single mode fibers SMF1 and SMF2. The illumination beam originates from SMF1, is polarized by filter LP1, collimated by lens L4, and is then directed via flip mirror FM1 and adjustable mirror KM3 to the sample (i.e. the "object"). This object wave is then relayed via lens L3, mirrors KM4 and FM2, and lens L2 onto the surface of the camera. The off-axis reference beam originates from the second fiber SMF2, is polarized by LP2, collimated by L5, relayed by mirrors KM1 and KM2, and superposed with the object beam through a 50:50 beam splitter BS. Mirrors KM1 and KM2 allow adjusting the off-axis angle of the reference beam. [1]

B. Results

In this first part of the experiment, the mirror FM1 is flipped down and FM2 is flipped up. FH3 holds a piece of paper to block the beam in order to avoid any accident. In this

configuration, only the beam coming from SMF2 is reaching the camera, which gives Figure 4. This is the reference wave as the object is placed on the other path (Sample in Figure 3).

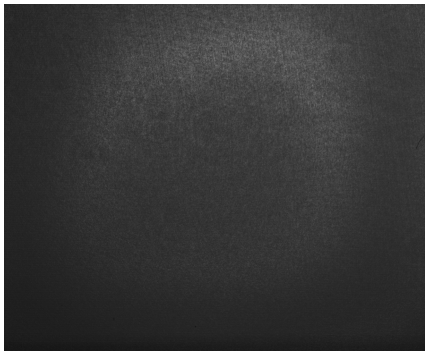

Fig. 4. Reference wave output

In order to observe something in the x, y plane, the exposure time is doubled (see Figure 5). Indeed, it allows to sense twice the quantity of incoming photons.

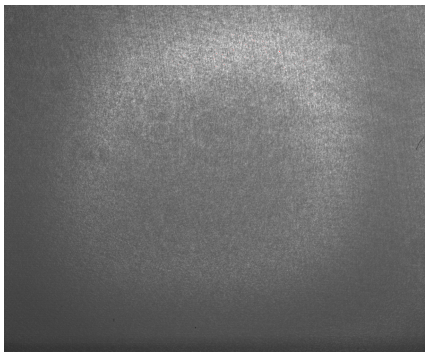

Fig. 5. Reference wave output with twice the exposure time

The output is also plotted in the Fourier plane in Figure 6, as it helps analyzing the results.

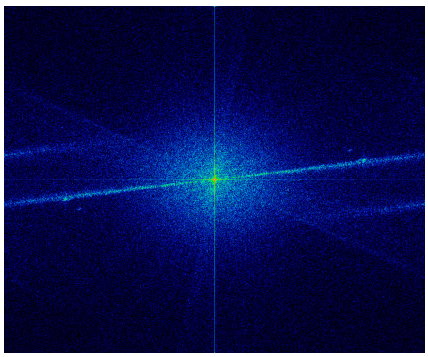

Fig. 6. Reference wave output in the Fourier plane

Now, FM2 is also flipped up to complete the path of the wave coming from SMF1, the object wave. Moreover, to eliminate the reference wave, a piece of paper is placed in FH2 (filter holder). This allow to only observe the object wave. It is shown in Figure 7.

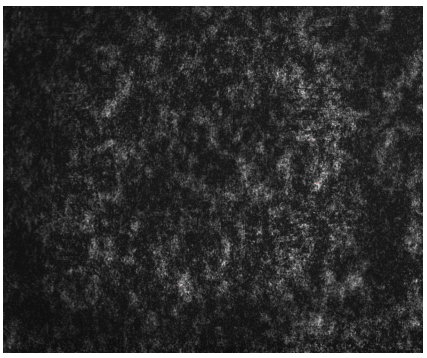

Fig. 7. Object wave output

Here also, the Fourier transform in taken to get a better appreciation of the result, see Figure 8.

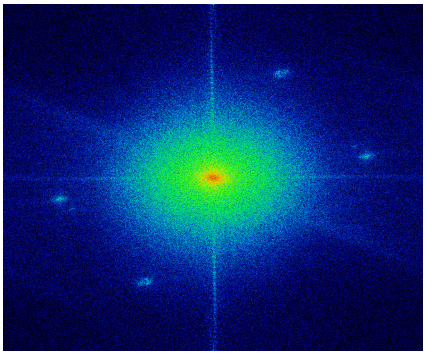

Fig. 8. Object wave output in the Fourier plane

Now that the profiles of both waves are known let us analyze the output when both waves attain the camera.

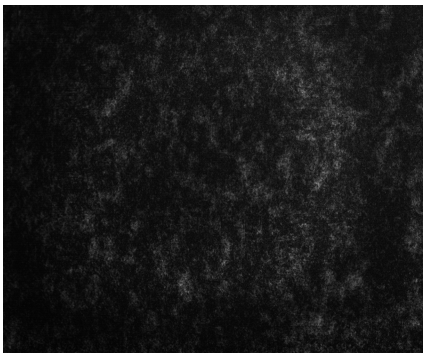

Fig. 9. Object and references waves output

The result (shown in Figure 9) is not surprising based on the theory developed in the first part of the report. In the Fourier plane (see Figure 10), there is a center information being the mix of the object and reference waves, and then there are two shifted areas representing the object and its conjugate. Unfortunately it is not possible to know which one is which

one because by changing the θ_x and θ_y, those areas move. In the Fourier plane, if you go to far to the left for example, you will disappear and reappear on the right side. The same statement can be made for going up or down.

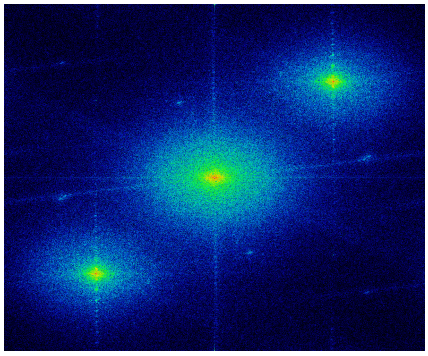

Fig. 10. Object and references waves output in the Fourier plane

For better analysis, the mirrors variables θ_x and θ_y are adjusted so that the object, its conjugate and the intensities of the object and reference wave do not interfere with each others. The final configuration is shown in Figure 10. This movement of the mirrors angles is done by turning the knobs on KM2.

The impact of the angles is shown in Figure 11. The notion of going from the right to the left side is also visible and it is easily understandable that this configuration is less interesting than the previous one. Indeed, the object and its conjugate are almost interfering. To get close to the center, the value of θ_x and/or θ_y has to get close to $0°$.

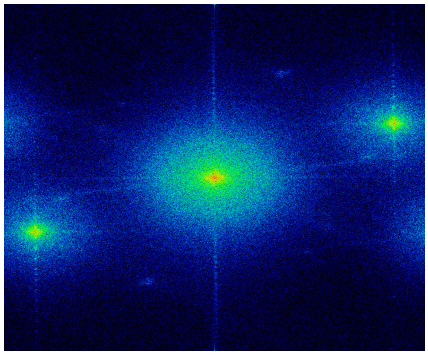

Fig. 11. Object and references waves output in the Fourier plane for other angles

There is also an impact in the spatial plane. When the values of θ_x and/or θ_y decrease, the sine becomes longer and longer. This results in long fringes. Note that if only θ_x decreases, the elongation of the sine will only be in the x direction so the spatial variables are independents.

IV. Task 3: Digitally reconstruct the off-axis hologram.

This task aims to reconstruct the off-axis hologram, based on the results of Task 3. By looking at the sample in the experimental setup by eye, we observe some sorts of stripes, but no big pattern is well visible. We will then analyse it more in details with Matlab. 48. The work will be to isolate the object wave based on Figure 10 and be able to identify what is drawn.

Figure 39 is simply the output seen by the camera. This image is first stored in `frame` variable, and transformed using the Fourier transform as done in the previous section.

```
figure(1)
imagesc(frame);
img=fft2s(frame);
```

Fig. 12. Matlab code for taking the Fourier transform of `frame` variable

This output is not very interesting because nothing is visible due to the scale of the Figure. In fact the information is concentrated on three very small spots and in order to see them a dB scale is applied ($log - log$).

```
figure(2)
imagesc(db(abs(img)));
```

Fig. 13. Matlab code for ploting `frame` in dB-scale

This transformation gives Figure 40 where the three spots are visible. In the center there ire I_R and I_O and the shifted information is the object and its conjugate (see theory). To isolate one of the two spots, a circular mask is created. This mask is centered in (286 ; 838) and has a radius of 183. The area selected is made of 1 and the rest of 0 as shown in Figure 41.

This mask multiplied by Figure 40 gives the output in Figure 42. This image is then centered by a translation of 369 in the x direction and -300 in the y direction using MATLAB function.

```
m=mask_circular([1082 1312],286,838,183);
figure(3)
imagesc(m);

figure(4)
img2=m.*img;
imagesc(db(abs(img2)));

figure(5)
image_shifted=move_xy(img2,369,-300);
imagesc(db(abs(image_shifted)));
```

Fig. 14. Matlab code for creating the mask, applying it and shifting it

The image was put in the Fourier domain but now it needs to get back to the spatial plane explaining the usage of the inverse Fourier transform to go from Figure 43 to 44.

The last step is to take the phase of this output image to observe the image "drawn" on the thin glass. This image shown in Figure 16 is actually the USAF51 used in optics to characterize the performances of an optical system. [5]

```
%% plots results
figure(6)
final=ifft2s(image_shifted);
imagesc(abs(final));

figure(7)
imagesc(angle(final));
```

Fig. 15. Matlab code for going back to the spatial plane

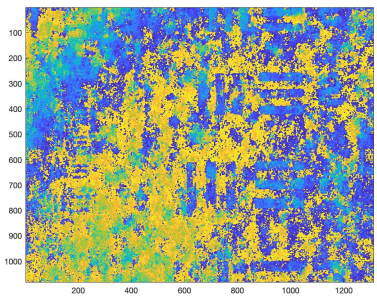

Fig. 16. Plot of the phase of Figure 44. Final result of Task 3.

The image is observed through the phase because the sample creates a phase delay due to different glass widths. It is such as a layer with the drawing was applied on the fin glass. In Figure 17, the phase ϕ_1 will be delayed compared to ϕ_2 due to the difference d and the change in the refractive index (n). The value of d can be computed.

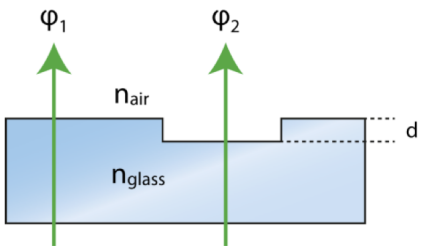

Fig. 17. Situation explaining the phase shifts

Equation (10) shows the relation between the phase difference of the two waves and the distance d. By isolating d, equation (11) is deduced.

$$\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{\lambda} d(n_{glass} - n_{air}) \tag{10}$$

The only unknown value is the phase difference between ϕ_1 and ϕ_2 as the indexes n_{air} and n_{glass} are respectively 1 and 1.45 and $\lambda = 532\ nm$. To find the phase difference, a value of the phase of a yellow point and a value the one of a blue point are taken in Figure 16. The two points selected are shown on Figures (45) and (46) in the Appendix and there values are next to the variable "Index". We have $\phi_{blue} = \phi_1 = -2.90$ $\phi_{yellow} = \phi_2 = 2.52$

$$(10) \implies d = \frac{\lambda\Delta\phi}{2\pi(n_{glass} - n_{air})} \tag{11}$$

This gives a distance $d = 1.02\ \mu m$. Another try was done by choosing two other point (less bright) and the distance d found then was $0.76\ \mu m$. If a mean was done on the points, the value found would be close to $0.8\ \mu m$.

V. Task 4: Spatial Light Modulator

This section aims to use a spatial light modulator (SLM) for changing the phase of an incoming light. The SLM is a device used for modulating the amplitude, the phase or the polarization of light. Is is often made of liquid and anisotropic crystals stuffed into two electrodes as shown in Figure 18. [6]

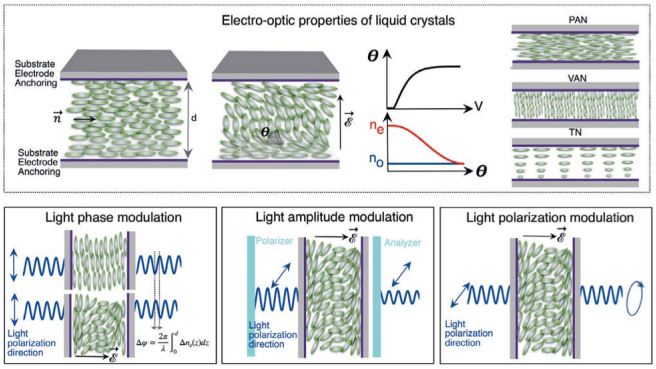

Fig. 18. Working principle of a SLM [7]

A. Setup

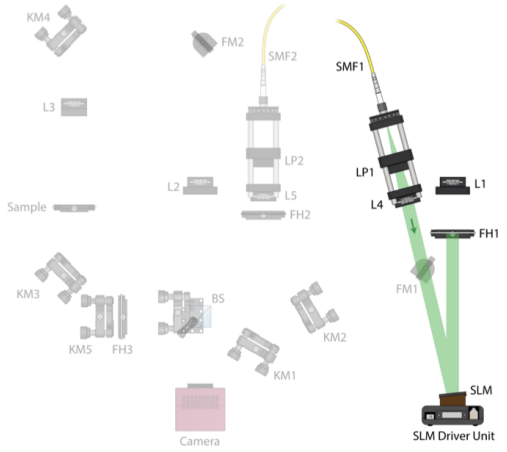

Fig. 19. Setup for the spatial light modulator

The setup for the SLM experiments is shown in Figure 19. The light beam coming out of fiber SMF1 is filtered by polarizer LP1, collimated by L4, and directed on the SLM. A viewing card can be placed in filter holder FH1 in order to observe the patterns reflected by the SLM. The specifications of the SLM are shown in Table I. The SLM is connected as a "second monitor" to the computer. To display phase distributions, it is necessary to open a window on this second monitor and display the phase as an 8-bit grayscale image (gray level 0 is phase $0°$*, gray level 128 is phase* $180°$*, etc.).* [1].

TABLE I
SLM SPECIFICATIONS

Variable	Value	Unit
Resolution	1920×1080	pixels
Pixel pitch	8	μm
Fill factor	93	%
Active area	15.36×8.64	mm
Bit depth	8 (256)	bits (gray levels)
Video frame rate	60	Hz

B. Results

Before doing anything, it is good to see what is obtained after the SLM. Figure 20 shows a rectangular aperture and some degeneration. This is simply the beam being reflected by a rectangular mirror. The mirror being in a rectangular shape explains the apparition of sinc functions.

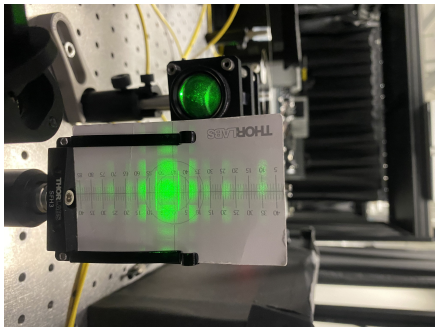

Fig. 20. Output of SLM when doing nothing

Note that all the following steps are indicated in Figure 48, which is the Matlab code for Task 4.

The first step of this section is to represent the SLM on Matlab. The grid is created using meshgrid function.

This creates two tables, one with one common value per row and the other with one common value per column. Between the rows and columns there is a spacing corresponding to the size of dimensions of the frame divided by the number of pixels. This gives a grid of square with side length of $8\mu m$ ($\frac{15.36\ mm}{1920} = \frac{8.64\ mm}{1080} = 8\ \mu m$).

The goal now is to simulate a lens following equation (12) with $f = 200\ mm$.

$$F = e^{j\frac{2\pi}{\lambda}\frac{(x^2+y^2)}{2f}} \tag{12}$$

The output after applying the phase mask is represented in Figure 21. The degenerations are still visible but this time the output is the focused light. Note that the holder is placed at $200\ mm$ from the SLM to be at the focal length. The SLM can thus simulate a lens but not perfectly. Indeed, in real life, the degeneration caused by the computer would not be visible.

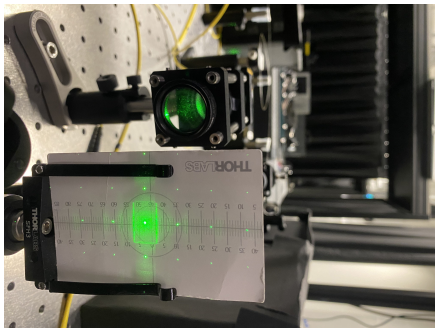

Fig. 21. Output of SLM after applying lens

The computation of the angular separation between two points is done based on the situation represented in Figure 22.

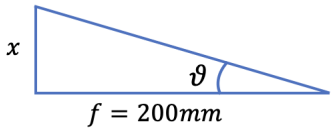

Fig. 22. Situation to compute angular separation

The angular separation is given by formula (9) where x is the spacing between two points on Figure 21.

$$\tan(\theta) = \frac{x}{f} = \frac{13}{200} \iff \theta = \arctan\left(\frac{13}{200}\right) = 3.72° \quad (13)$$

The degeneration is caused by the SLM and especially by the crystals inside it. Each crystal can modulate the phase of a pixel. Those crystals are periodic and based on the angular distance, it is possible to compute the period T of the lattice. This is done using formula (11) from the theoretical background (Task 1). The period found is approximately the size of one pixel (true in both direction).

$$T = \frac{532\ nm}{\sin(\theta)} = \frac{532\ nm}{\sin(3.72°)} = 8201\ nm = 8.2\ \mu m \quad (14)$$

The error made is less than 3% so it is relatively acceptable. Note that if $x = 8\ \mu m$, $\theta = 2.29°$.

If the SLM was turned off, the diffraction would still occur since the different orders we can observe are due the periodic structure of the SLM mirrors.

The next paragraph is consecrated to a study of the output when the focal length is changed.

It is interesting to try various focal lengths for the lens. It is done and the images obtained with SLM are shown in Figures 23 and 24.

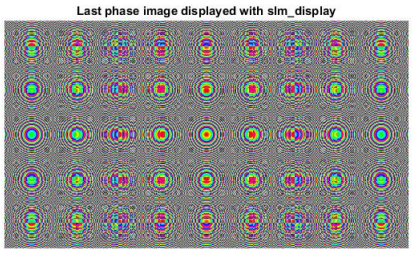

Fig. 23. Image obtained with SLM for $f = 100\,\mathrm{mm}$

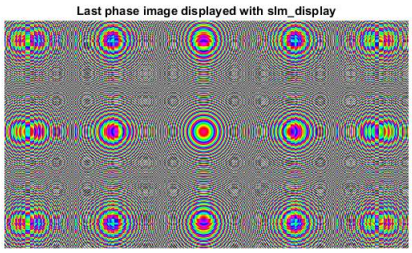

Fig. 24. Image obtained with SLM for $f = 1000\,\mathrm{mm}$

We see on these figures that the focal points do change when we play with f, in particular, a high f gives a long distance to the focal plane.

A linear mask will now be applied by the SLM to see the impact after a propagation of 200 mm. The mask is defined by equation (15). The values p_x and p_y represent a translation vector in the spatial plane but here in the Fourier plane its units are m^{-1}.

$$F = e^{j2\pi(p_x x + p_y y)} \quad (15)$$

To analyze the behaviour, different values of p_x and p_y are tried between 1000 m^{-1} and 100000 m^{-1}.

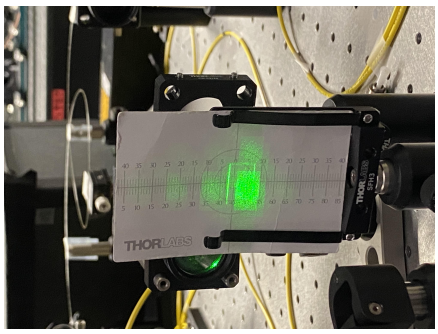

Fig. 25. Output of SLM after translation mask applied

It is clear that the rectangular aperture is shifted in both x and y directions. The intensity of the shift is linked to the value of p_x and p_y.

With Figures 26 and 27, one can understand the influence of the values of p_x and p_y, the parameters of the mask. These values change the angle of the stripes in the figure of diffraction. The bigger p_x and p_y are, the more horizontal the stripes will be. In fact, the phase seems to be stripes because it is defined modulo 2π, which implies when reaching an angle of 2π, it goes to $zero$.

Fig. 26. Image obtained with SLM with $p_x = p_y = 1000\,\mathrm{m}^{-1}$

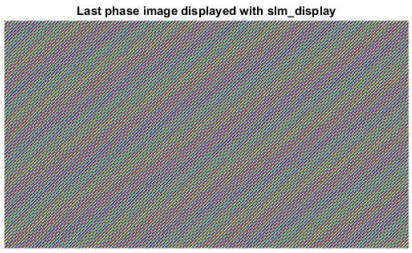

Fig. 27. Image obtained with SLM with $p_x = p_y = 100\,000\,\mathrm{m}^{-1}$

By combining the lens and the translation mask previously used, the output becomes Figure 28. It is simply a combination of the two effects. The light is focused at a distance $f = 200\ mm$ and translated by p_x,p_y.

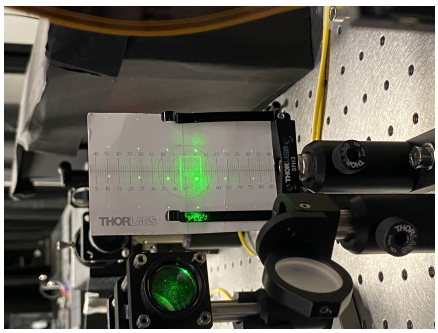

Fig. 28. Output of SLM after lens and translation mask applied

When the light is displayed on the SLM not all of it is modulated. Indeed, by looking closely to Figure 28, the rectangular shape and the high orders points are still visible. There is a spacing between two crystals and light can arrive on this particular type of spots. This implies that the light won't be modulated but just reflected.

The last step of this experiment is to run the code `slm_demo.m`. This displays the former EPFL logo at a focal length of $f = 200\ mm$. By moving the paper representing the camera plane, the EPFL logo becomes a smiley. This kind of pattern is created using back propagation (BPM theory).

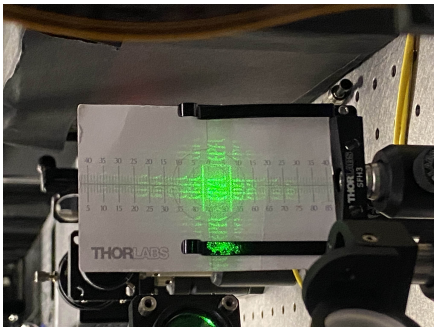

Fig. 29. EPFL logo obtained by running `slm_demo.m` 49

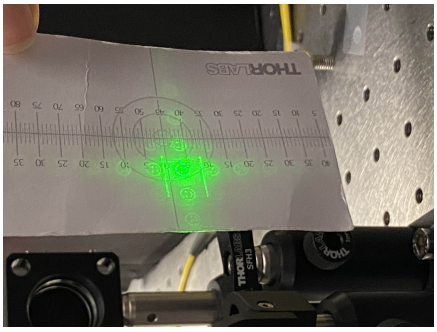

Fig. 30. Smiley obtained by back propagation of the EPFL logo

VI. Task 5: Numerical simulation of a matrix measurement

In this section, the notion of transmission matrix is studied. First a theoretical approach will be done and after the SLM output will be studied. All the lines of code of this section are extracted from `task5.m` (see Figure 50). To begin with, here are the input and output for a test example.

$$X_{test} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}; Y_{test} = \begin{pmatrix} 1+7i & 1+3i \\ -3-i & -3+3i \end{pmatrix} \tag{16}$$

```
x1=[1;1i];
x2=[1;-1i];
X_test=[x1 x2];

y1=[1+7*1i;-3-1i];
y2=[1+3*1i;-3-3i];
Y_test=[y1 y2];
```

Fig. 31. Matlab code for setting X_{test} and Y_{test}

The system has this form : $Y = T \cdot X$. By applying this form to the test samples, the transmission matrix can be found using $T = X_{test}^{-1} \cdot Y_{test}$
After performing the matrix product, the transmission matrix is found. T is written here under.

$$T = \begin{pmatrix} 5i & 2+3i \\ 1+2i & -1 \end{pmatrix} \tag{17}$$

```
T=inv(X_test)*Y_test
```

Fig. 32. Matlab code for computing T

The purpose now is to find the input if the same transmission matrix was used and the output is the following matrix. Because the output matrix is 2×1, the input has the same dimensions.

$$Y = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \tag{18}$$

The principle stays the same but this time the variable to isolate is X. This gives $X = T^{-1} \cdot Y$

$$X = \begin{pmatrix} 5i & 2+3i \\ 1+2i & -1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} -0.6+0.45i \\ 1.5-0.75i \end{pmatrix} \tag{19}$$

```
y3=[3;-3];
x3=inv(T)*y3;

n=norm(x3)
p=angle(x3)

polar=exp(1i*p);
y_polar=T*polar;
abs(y_polar);
```

Fig. 33. Matlab code for computing X knowing Y

The question now is: can the pattern computed just above be displayed by the SLM ? The answer is no because the SLM can only act on the phase. If we set the amplitude of each pixel to 1, we would obtain with a phase only input the vector $\begin{pmatrix} 1.472 \\ -1.472 \end{pmatrix}$.

VII. Task 6: Experimental matrix measurement

This section will be about the impact on the output due to slight modifications on the fiber.

A. Setup

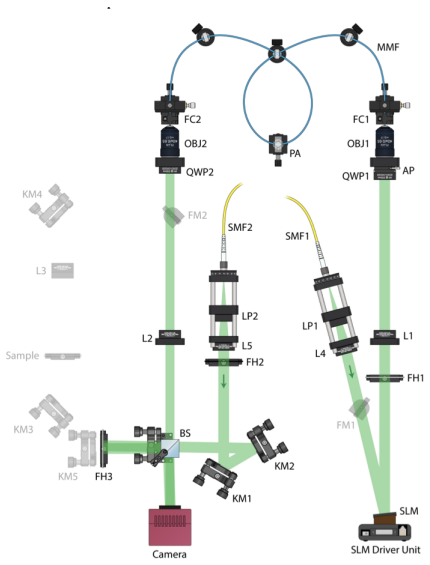

Fig. 34. Setup for experimental matrix measurement

The experimental setup for the transmission matrix is shown in Figure 34. Light originates from SMF1, is collimated by L4 and then directed to the SLM. The pattern created by the SLM is imaged through the relay system composed of lenses L1 ($f = 250\ mm$) and OBJ1 ($f = 3\ mm$), with a demagnification factor of approximately 83. This pattern then travels through the multimode fiber MMF, whose tip is fixed in fiber clamp FC1. A quarter-wave plate QWP1 changes the polarization to circular, as this polarization is better preserved in step-index fibers. The aperture AP is used to block unwanted extra diffraction orders from the SLM. In the middle of the fiber, an adjuster PA allows to slightly change the bending state of the fiber (see later). At the other end of the fiber, an identical relay system OBJ2 and L2 magnifies the fiber image and projects it on the camera, with quarter wave plate QWP2 converting the polarization back to linear. The off-axis reference beam originates from SMF2, is linearly polarized by LP2, collimated by L5, and superposed to the fiber image on the camera via beam splitter BS. Filter holder FH2 can be used to temporarily block the reference beam, and the viewing card in FH3 can be used to directly visualize the fiber output without the aid of the camera [1].

B. Results

First of all, the specifications of the fiber are described. The core diameter is $50\ \mu m$ and the numerical aperture is 0.22. Based on this data and using Equation (20), the V number of the fiber is deduced.

$$V = \frac{2\pi}{\lambda} rNA = 64.96 \tag{20}$$

With the V number, the number of modes M can be approximated according to Equation (21).

$$M \approx \frac{V^2}{4} = \lfloor 1054.88 \rfloor = 1054 \text{ modes} \tag{21}$$

In order to record all the modes, a square matrix of size 33×33 is needed ($33^2 = 1089$). If each element of the matrix is of size 16 bytes, the total number of bytes is 16864. If the size of the diameter or the numerical aperture doubled, then the V number doubled and the number of modes is multiplied by 4 so does the number of bytes. The number of modes becomes 4219 and therefore the number of bytes 67504.

In order to characterize the impact of the fiber on the output of the SLM, a first recording is done. Indeed, a multimode fiber does not transmit data perfectly but scrambles everything as shown on Figure 35.

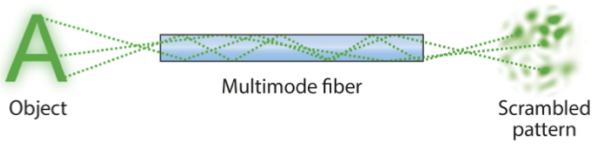

Fig. 35. Behaviour of multimode fiber

The first recording appeared on the screen and an area containing the information was taken. This area defines the time of computation because the rest of the picture is ignored. This selected zone is transformed in the Fourier domain, and marked with colors to distinguish the conjugate from the real order. As done previously, the part used to measure the impact of the transmission matrix is the real order. The conjugate could in principle be used but it would be much complicated. Once the part is isolated, a bunch of test images are sent and the outputs are recorded. This allows the computer to create T, the transmission matrix. During the process, the fiber has to stay as motionless as possible otherwise it would change the transmission matrix. Finally, images can be sent by the SLM into the fiber and the output recorded can be adjust by the previous transmission matrix in order to reconstruct the hologram sent. To better observe the output, the reference beam is blocked.

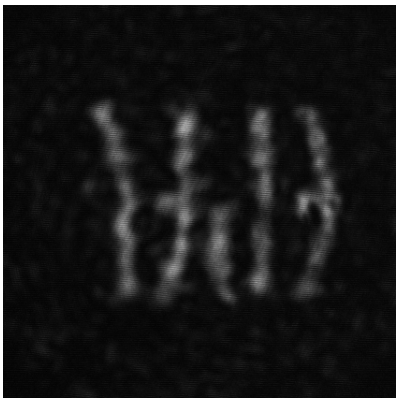

Fig. 36. EPFL logo recorder at output of the fiber

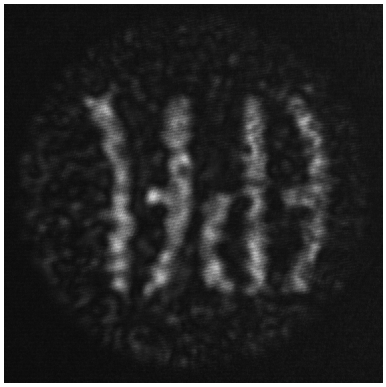

Fig. 37. EPFL logo recorder at output of the fiber (twice the exposure)

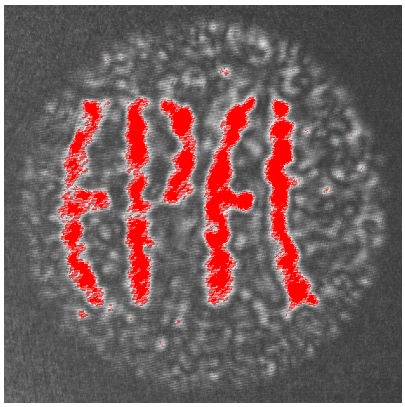

Fig. 38. EPFL logo recorder at output of the fiber ($4\times$ the exposure)

To better observe the image, it is sometimes a good idea to have a longer exposure time. This exposure time can't be too long otherwise, some pixels might get over exposed. This is shown by red pixels on the output image. In this case, the experiment was done with overexposure as only the pixels of the logo were becoming red helping visualising it. In real life this wouldn't be done.

The last observation done during the laboratory is the study of modifying the fiber by bending it. First of all, when touched with hands, the pattern at the output immediately changes. To understand the precision of a fiber, a PA is used. It bends the fiber by small distances allowing a precise study. In fact one revolution corresponds to a shift in the axis of $0.35\,\mathrm{mm}$. Before obtaining an unrecognizable image, 3.75 revolutions were performed, and the fiber was therefore bend by $1.3\,\mathrm{mm}$. When moving the fiber back to its initial position, the pattern reappears. This means that the fiber was conceived in a way that bending would not impact the structure. If it did when coming back to the initial position, the fiber would not reconstruct the pattern.

VIII. Conclusion

This laboratory was interesting because it made us improve two types of skills: practical and theoretical ones.

First, during the preparation, we had the opportunity to learn more about the theory behind digital holography: its working principle, its optical components and the way they shall be disposed to perform holography, and one of its applications for measuring the width of a layer. We also could discover the SLM technology which is a device we could integrate in another setup for a complete different application. Finally, we learned a lot about transmission in a multimode fiber, which can be characterized by a matrix T and altered by changing some parameters of the fiber.

In a practical manner, it is always satisfying for engineering students to touch and sense their object of study. This laboratory session with Leo, gave us the opportunity to have a good first appreciation of the construction and the use of digital holography.

References

[1] Damien Loterie, Maya Konstantinou, *Optics Laboratory Work: Digital Holography* [PDF]

[2] Ting-Chung Poon, Jung-Ping Liu, *Introduction to modern holography* [Internet] https://www.cambridge.org/core/books/abs/introduction-to-modern-digital-holography/applications-in-digital-holography/C5C520DC3DC5F8056ACD2D54BFA4020

[3] Prof. Olivier Martin, *Polycopié d'Ingénierie Optique - Micro-321 @EPFL* [Course notes]

[4] Wikipedia (22th May, 2022) *Holography* [Internet] https://en.wikipedia.org/wiki/Holography

[5] Edmund Optics *1951 USAF Resolution Calculator* [Internet] https://www.edmundoptics.com/knowledge-center/tech-tools/1951-usaf-resolution/

[6] Holoeye *Spatial Light Modulators* [Internet] https://holoeye.com/spatial-light-modulators/

[7] Aurélien Jullier, Physics Institute of Nice (France) *Spatial Light Modulators Course* [Internet] https://www.photoniques.com/articles/photon/pdf/2020/02/photon2020101p59.pdf

IX. Appendix

A. Matlab plots

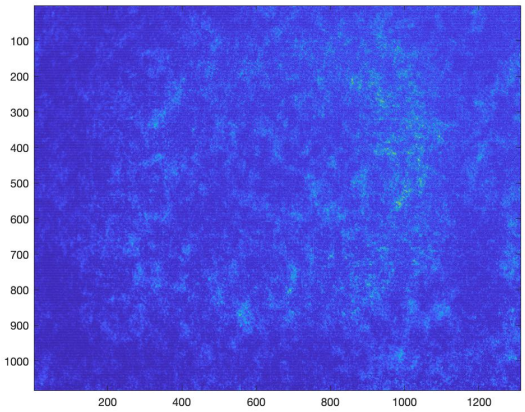

Fig. 39. Output of camera

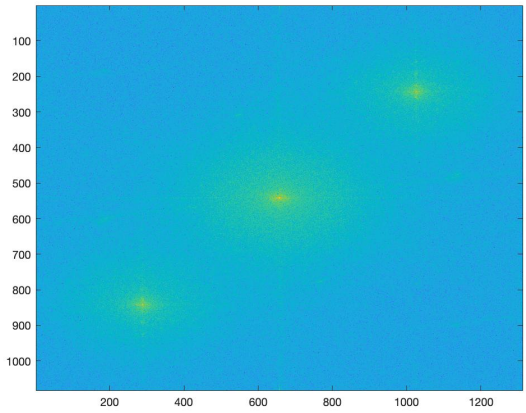

Fig. 40. Output of camera in log scale after Fourier transform

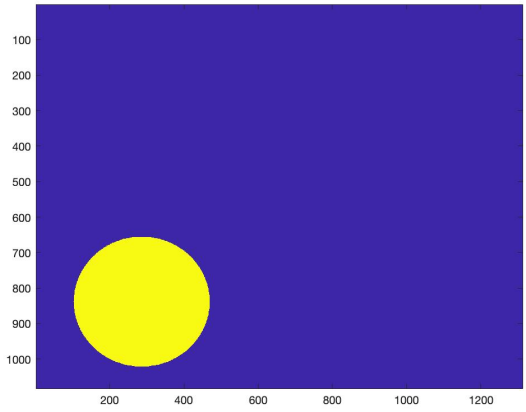

Fig. 41. Created mask

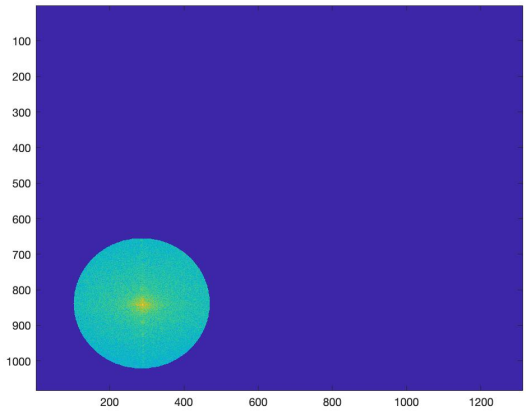

Fig. 42. Mask applied on the output in log scale

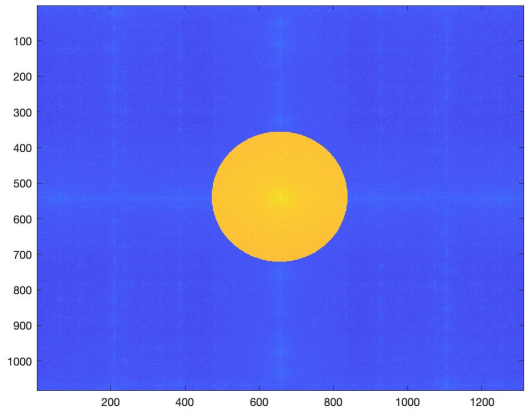

Fig. 43. Information centered

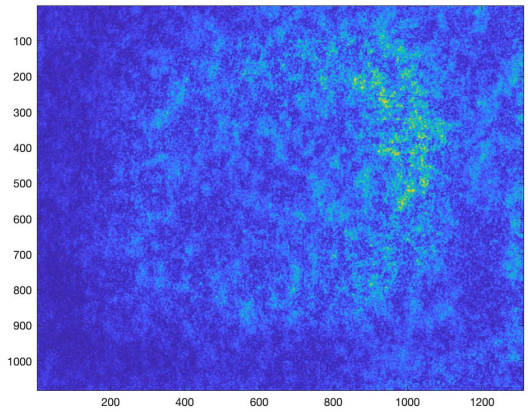

Fig. 44. Plot of the output after inverse Fourier transform

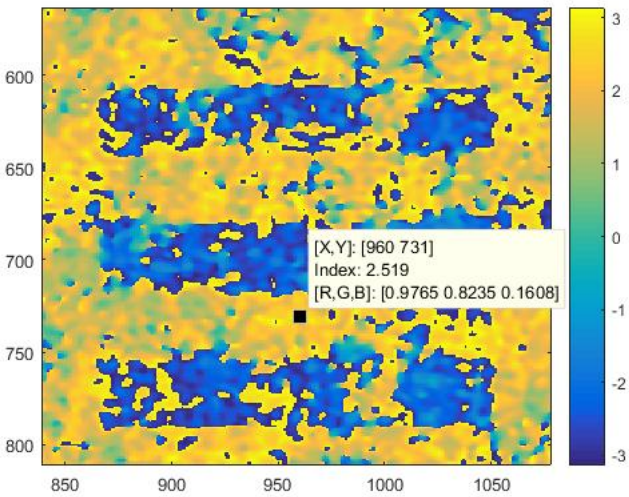

Fig. 45. Blue point for computation of d

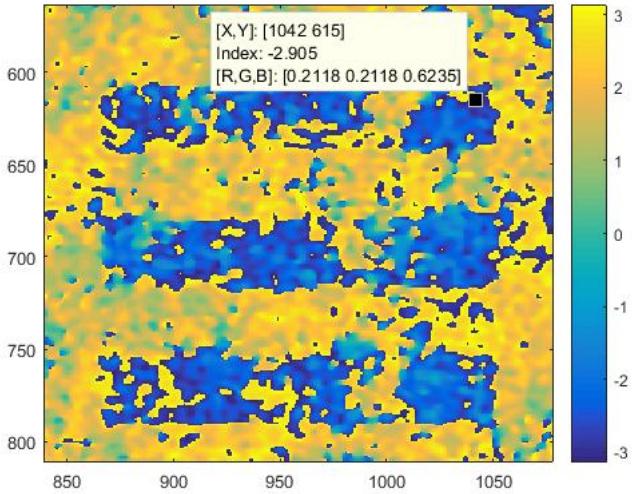

Fig. 46. Yellow point for computation of d

B. Matlab scripts

```
% TP holography - TASK 3
% 12th, May 2022
% Group 4

clear all;
close all;
clc;

load('r_adjustedangle_fourier.m');

figure(1)
imagesc(frame);
img=fft2s(frame);

figure(2)
imagesc(db(abs(img)));

m=mask_circular([1082 1312],286,838,183);
figure(3)
imagesc(m);

figure(4)
img2=m.*img;
imagesc(db(abs(img2)));

figure(5)
image_shifted=move_xy(img2,369,-300);
imagesc(db(abs(image_shifted)));

%% plots results
figure(6)
final=ifft2s(image_shifted);
imagesc(abs(final));

figure(7)
imagesc(angle(final));
```

Fig. 47. Matlab code for Task 3

```
% TP holography - TASK 4
% 12th, May 2022
% Group 4

clear all;
close all;
clc;

%% lens phase mask
% create meshgrid
x = -540:1:539;
y = -960:1:959;
x = x*8e-6;
y = y*8e-6;
[Y,X] = meshgrid (y, x);

% create a lens
f = 200e-3;
lambda = 532e-9;
F_lens = exp( 1i *(2* pi *(X.^2+Y.^2) ) /( lambda *2* f ) ) ;
slm_display(F_lens);

%% show image of F_len in MATLAB
clims = [0 20];
imagesc(x,y,angle(F_lens),clims);
set(colorbar,'FontName','Times New Roman','FontSize',20,'LineWidth',1)
set(gca,'FontName','Times New Roman','FontSize',20,'LineWidth',1)
xlabel('x coordinate [\mum]')
ylabel('y coordinate [\mum]')
title('phase mask');
axis image;
theta = atan(13/200);
d = lambda/(sin(theta));

%% linear phase mask
px = 0; % vertical - select between 1e3 to 1e5
py = 0; % horizontal - select between 1e3 to 1e5
F_lin = exp ( 1i *(2* pi *( px*X+py*Y) ) ) ;
slm_display(F_lin)

%% combined phase mask
F_com = F_lin.*F_lens;
% slm_display(F_com)
```

Fig. 48. Matlab code for Task 4

```
% Check for SLM object
if ~exist('slm','var')
   slm = dx_fullscreen();
end
x_slm = slm.getConfig('frameWidth');
y_slm = slm.getConfig('frameHeight');

% Smiley at f = 100mm
img1 = double(max(imread([tp_folder_path '/tm/img/smiley.png']),[],3));
img1 = imresize(img1,2);
img1 = pad_center(img1,[y_slm, x_slm]);
img1 = img1.*exp(2i*pi*rand(size(img1)));
img1 = propagate(img1, 100e-3);

% EPFL at f = 200mm
img2 = double(max(imread([tp_folder_path '/tm/img/epfl.png']),[],3));
img2 = img2(:,end:-1:1);
img2 = imresize(img2,4);
img2 = pad_center(img2,[y_slm, x_slm]);
img2 = img2.*exp(2i*pi*rand(size(img2)));
img2 = propagate(img2, 200e-3);

% Display both
slm_display(img1+img2);
```

Fig. 49. `slm_demo.m`: Matlab code for the last part of Task 4

```
% TP holography - TASK 5
% 12th, May 2022
% Group 4

clear all;
close all;
clc;

x1=[1;1i];
x2=[1;-1i];
X_test=[x1 x2];

y1=[1+7*1i;-3-1i];
y2=[1+3*1i;-3-3i];
Y_test=[y1 y2];

T=inv(X_test)*Y_test
%linsolve(y1,x1)
%mldivide(y1,x1)

y3=[3;-3];
x3=inv(T)*y3;

n=norm(x3)
p=angle(x3)

polar=exp(1i*p);
y_polar=T*polar;
abs(y_polar);

x_new=x3/n;
y_new=T*x_new;
```

Fig. 50. Matlab code for Task 5